



Magnetic Fields



Magnetism

- Magnets can exert forces on each other.
- The magnetic forces between north and south poles have the property that like poles repel each other, and unlike poles attract.
- This behavior is similar to that of like and unlike electric charges.



Magnetism cont.

- However, there is a significant difference between magnetic poles and electric charges.
- It is possible to separate positive from negative charges but no one has ever been able to do so with the north and south poles of a magnet.
- There appears to be no existence of magnetic monopoles.



Magnetic Forces

- When a charge is placed in an electric field it experiences an electric force.
- It is natural to ask whether a charge placed in a magnetic field experiences a magnetic force.
- The answer is yes provided two conditions are met:



Magnetic Forces cont.

- 1. The charge must be moving.
- 2. The velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field.



Magnetic Forces on Charges

- The form of the magnetic force on a moving charged particle is given by the following:

$$\vec{F} = q_0 \vec{v} \times \vec{B}$$

- Here v represents the velocity of the particle, q_0 is the charge of the particle, and B is the magnetic field.



Magnetic Forces on Charges

- The magnitude of the force can be obtained by using the definition of the vector cross product.
- The angle θ is the angle between v and B .

$$F = q_o v B \sin \theta$$



Magnitude of the B-Field

- The magnitude of the magnetic field can now be defined similar to that of the electric field by:

$$B = \frac{F}{q_o v \sin \theta}$$



Direction and Units

- The magnetic field is a vector and its direction can be determined using a compass or the right hand rule of vector multiplication.
- The unit of magnetism is:

$$\frac{N \cdot s}{C \cdot m} = \textit{tesla} (T)$$



Example:

- A proton in a particle accelerator has a speed of 5.0×10^6 m/s.
- The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes an angle of 30.0° with respect to the proton's velocity.
- Find the magnitude and direction of the magnetic force on the proton.



Solution

- The positive charge on a proton is $1.60 \times 10^{-19} \text{ C}$, therefore, the magnetic force acting on the proton is:

$$F = q_o v B \sin \theta$$



Solution cont.

- Plugging in our values we get the following:

$$F = (1.6 \times 10^{-19} \text{ C})(5 \times 10^6 \text{ m/s})(0.40 \text{ T})(\sin 30.0^\circ)$$
$$= 1.6 \times 10^{-13} \text{ N}$$



Example

- In an attempt to catch the Road Runner, Wile E. Coyote suspends a powerful electromagnet from a very high cliff overlooking a road that is frequented by the Road Runner.
- A small pile of iron laced bird seed is placed directly beneath the magnet in hopes that the RoadRunner will eat the food and then be pulled up by the magnet once it is turned on.



Example cont.

- As usual things go wrong and the Coyote is unable to turn the magnet on before the RoadRunner escapes.
- In addition, the magnet slips from its perch and lodges itself halfway down the cliff on its side.
- In his attempt to activate the magnet the Coyote slides down the hill he is sitting on and falls off the edge of the cliff where the magnet is suspended.



Example cont.

- During his slide down the hill the Coyote acquires a static charge of $6500 \mu\text{C}$ to his fur.
- Once he reaches terminal velocity, 54 m/s , the magnet is activated.
- Assume that the Coyote falls a distance of 7.0 m through the magnetic field and perpendicular to it. The field produced by the magnet is 5.0 T and the Coyote's mass is 42 kg .

Example cont.

- After leaving the magnetic field, the Coyote continues to fall 25 m more before striking the ground.
- How far from the center of the road will the Coyote strike the ground?





Solution

- In order to obtain the horizontal displacement of Wile E. we must determine the duration and magnitude of the magnetic force placed on him by the magnet.

$$F = qvB \sin \theta$$



Solution cont.

- The charge is $6500 \mu\text{C}$, his speed is 54 m/s , the magnetic field is 5.0 T , and the angle between the Coyote's velocity and the magnetic field is 90° .
The force is

$$F = (6500 \times 10^{-6} \text{ C})(54 \text{ m/s})(5.0 \text{ T}) = 1.8 \text{ N}$$



Solution cont.

- To obtain the displacement we need to determine the acceleration and the time in which the Coyote undergoes this acceleration.

$$a = \frac{F}{m} = \frac{1.8 N}{42 kg} = 4.3 \times 10^{-2} m / s^2$$



Solution cont.

- The time that the Coyote spends in the magnetic field is equal to the vertical distance traveled through the field divided by his speed.

$$t = \frac{d}{v} = \frac{7.0 \text{ m}}{54 \text{ m/s}} = 0.13 \text{ s}$$



Solution cont.

- The horizontal displacement travel while in the magnetic field can be obtained using Newton's equations of motion.

$$x = v_0 t + \frac{1}{2} a t^2$$

$$x = 0 + \frac{1}{2} (4.3 \times 10^{-2} \text{ m/s}^2) (0.13 \text{ s})^2$$

$$x = 3.6 \times 10^{-4} \text{ m}$$



Solution cont.

- To determine the total horizontal displacement we need to calculate the final horizontal velocity.

$$v_{f_x} = v_{0_x} + a_x t$$

$$v_{f_x} = 0 + (4.3 \times 10^{-2} \text{ m} / \text{s}^2)(0.13 \text{ s})$$

$$v_{f_x} = 5.6 \times 10^{-3} \text{ m} / \text{s}$$



Solution cont.

- The horizontal displacement that Wile E. travels once he leaves the magnetic field is equal to his horizontal speed times the time it takes him to fall to the ground.





Solution cont.

- We can calculate the time of his ascent by dividing the distance of his fall outside of the magnetic field by his constant speed of 54 m/s.

$$t = \frac{d}{v} = \frac{25 \text{ m}}{54 \text{ m/s}} = 0.46 \text{ s}$$



Solution cont.

- His horizontal displacement while outside of the magnetic field is then:

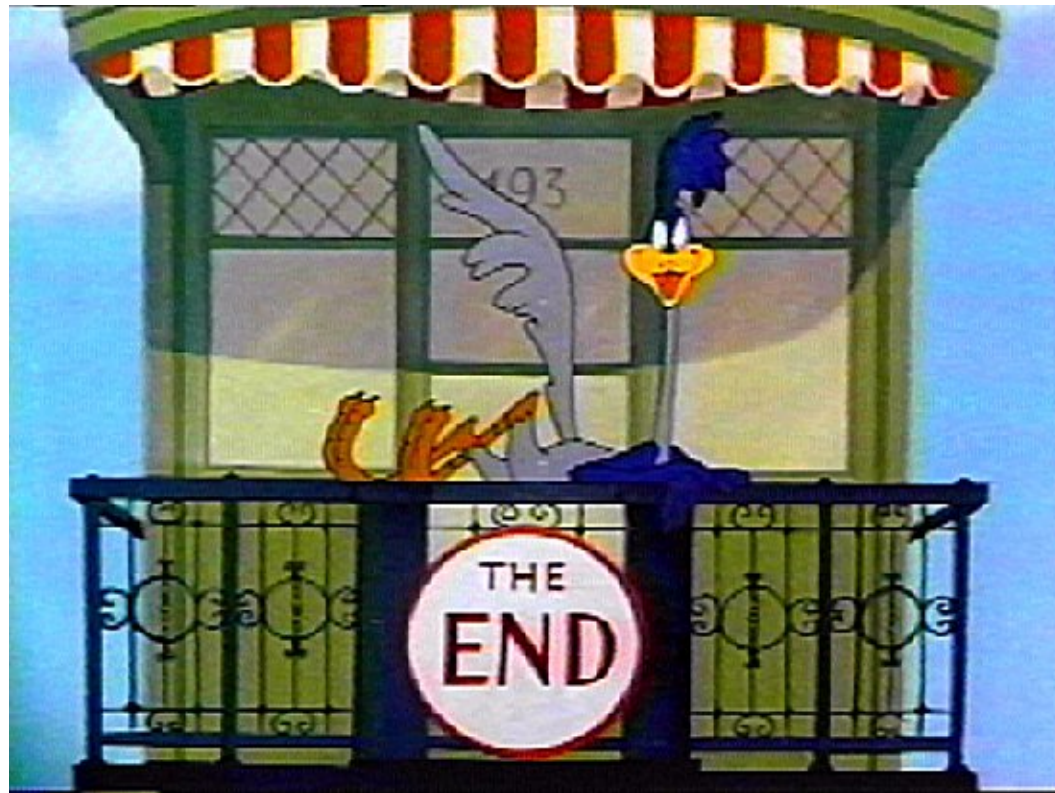
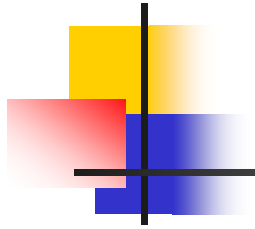
$$x = v_x t = (5.6 \times 10^{-3} \text{ m / s})(0.46 \text{ s}) = 2.6 \times 10^{-3} \text{ m}$$



Solution cont.

- The total horizontal displacement is then the sum of the horizontal displacement while he was in the magnetic field and the horizontal displacement while outside of the magnetic field.

$$x_{total} = 3.6 \times 10^{-3} m + 2.6 \times 10^{-3} m = 6.2 \times 10^{-3} m$$





Work Done on a Moving Charged Particle

- The work done by the force that an electric field applies on a charged particle is equal to the magnitude of the electric field multiplied by the charge on the particle and the distance that the particle moves in the direction of the applied force.



Moving Charge in an E-Field

- The work on a moving charge in an electric field can be written as:

$$W = q_0 \int \vec{E} \cdot d\vec{l} = q_0 \int E dl \cos \theta$$



Charge Moving in a B-Field

- The work done on a moving charged particle by a magnetic field is equal to zero because the magnetic force is always perpendicular the direction of motion.



Trajectory of a Moving Charge in a B-Field

- The magnetic force always acts perpendicular to the velocity of the charged particle and is directed toward the center of the circular path of the particle.
- Thus the force is a centripetal one.

$$F_c = m \frac{v^2}{r}$$



Trajectory of a Moving Charge in a B-Field cont.

- The magnitude of the magnetic force is given by:

$$q_o v B \sin 90^\circ = q_o v B$$



Trajectory of a Moving Charge in a B-Field cont.

- Therefore, by equating the two equations we get the following:
- The radius of the path of a particle is inversely proportional to the magnitude of the magnetic field.

$$q_o v B = \frac{mv^2}{r}$$

$$\Rightarrow r = \frac{mv}{qB}$$



Example

- A proton is released from rest at a point which is located next to the positive plate of a parallel plate capacitor.
- The proton then accelerates toward the negative plate, leaving the plate through a small hole in the capacitor.



Example cont.

- The electric potential of the positive plate is 2100 V greater than the negative plate.
- Once outside of the capacitor, the proton encounters a magnetic field of 0.10 T. The velocity is perpendicular to the magnetic field.
- Find the speed of the proton when it leaves the capacitor, and the radius of the circular path on which the proton moves in the magnetic field.



Solution

- The only force that acts on the proton while it is between the capacitor plates is the conservative electric force. Thus:

$$\frac{1}{2}mv_o^2 + qV_o = \frac{1}{2}mv_f^2 + qV_f$$



Solution cont.

- If we note that the initial velocity is zero and that the charge of the proton is equal in magnitude to that of the electron we can write the following:

$$\frac{1}{2}mv_f^2 = e(V_o - V_f)$$



Solution cont.

- Solving for the speed we get:

$$v_f = \sqrt{2e(V_o - V_f) / m} = \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(2100 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$v_f = 6.3 \times 10^5 \text{ m/s}$$



Solution cont.

- When the proton moves in the magnetic field, the radius of the circular path is:

$$r = \frac{mv_f}{eB} = \frac{(1.67 \times 10^{-27} \text{ kg})(6.3 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.10 \text{ T})}$$

$$r = 6.6 \times 10^{-2} \text{ m}$$



The Force on a Current in a Magnetic Field

- As we have seen, a charge moving through a magnetic field can experience a magnetic force.
- Since an electric current is a collection of moving charges, a current in the presence of a magnetic field can also experience a magnetic force.



Force on a Current cont.

- The magnitude of a magnetic force on a moving charge is given by:

$$F = qvB \sin \theta$$



Force on a Current cont.

- If we have a current carrying wire of length L and we envision a small differential increment of the wire, we see that the charge per unit time passing this segment of the wire can be expressed as:

$$\frac{dq}{dt}$$



Force on a Current cont.

- The length of wire traversed by the charge in the length of time, dt , is equal to the velocity of the charges multiplied by the time of travel:

$$L = v dt$$



Force on a Current cont.

- The force on the wire can now be deduced from the equation for the force on a single charge:

$$F = \frac{dq}{dt} v dt B \sin \theta$$



Force on a Current cont.

- Note: dq/dt is the definition of current.
- Therefore:

$$F = ILB \sin \theta$$



Force on a Current cont.

- As in the case of a single charge traveling in a magnetic field, the force is maximum when the wire is oriented perpendicular to the magnetic field, and vanishes when the current is parallel or antiparallel to the field.



Example

- The voice coil of a speaker has a diameter of 0.025 m, contains 55 turns of wire, and is placed in a 0.10 T magnetic field.
- The current in the voice coil is 2.0 A.
- Determine the magnetic force that acts on the coil and cone.
- If the coil and cone have a combined mass of 0.020 kg, find their acceleration.



Solution

- We can use the equation for a force on a current carrying wire.

$$F = ILB \sin \theta = (2.0A)[55\pi(0.025m)](0.1T)\sin 90^\circ$$

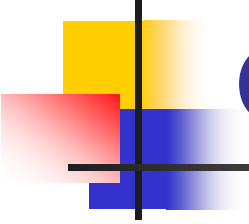
$$F = 0.86 N$$



Solution cont.

- According to Newton's second law:

$$a = \frac{F}{m} = \frac{0.86 \text{ N}}{0.020 \text{ kg}} = 43 \text{ m/s}^2$$



The Torque on a Current Carrying Coil

- If a loop of wire is suspended properly in a magnetic field, the magnetic force produces a torque that tends to rotate the loop.
- When a current carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field.



Torque on a Loop

- The expression for the torque on any flat loop of area A of any shape is given by the following:

$$\vec{\tau} = I \vec{A} \times \vec{B}$$



Torque on a Loop cont.

- If we define a magnetic moment of the loop in terms of the area vector and the current in the loop, then the torque becomes:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



Magnitude of the Torque

- We can write the magnitude of the torque created by a magnetic field on a current carrying loop as:

$$\tau = IAB \sin \theta = \mu B \sin \theta$$



Example

- A coil of wire has an area of $2 \times 10^{-4} \text{m}^2$, consists of 100 loops or turns, and contains a current of 0.045 A. The coil is placed in a uniform magnetic field of magnitude 0.15 T.
- Determine the maximum torque that the magnetic field can exert on the coil.



Solution

- Since the coil consists of numerous loops we need to modify our equation for torque.

$$\tau = NIAB \sin \theta$$

$$\tau = (100)(0.045 \text{ A})(2 \times 10^{-4} \text{ m}^2)(0.15 \text{ T})$$

$$\tau = 1.4 \times 10^{-4} \text{ Nm}$$



Example

- A rectangular coil of dimensions 5.40 cm by 8.50 cm consists of 25 turns of wire.
- The coil carries a current of 15 mA. A uniform magnetic field of magnitude 0.350 T is applied parallel to the plane of the loop.
- What are the magnitudes of the magnetic moment of the coil and the torque acting on the loop?



Solution

- The area of the rectangular coil is equal to the products of its length and height.

$$A = (0.0540 \text{ m})(0.085 \text{ m}) = 4.59 \times 10^{-3} \text{ m}^2$$



Solution cont.

- The magnitude of the magnetic moment is given by:

$$\mu = INA = (25)(15 \times 10^{-3} \text{ A})(4.59 \times 10^{-3} \text{ m}^2)$$

$$\mu = 1.72 \times 10^{-3} \text{ J / T}$$



Solution cont.

- The torque on the coil can be obtained from the following definition:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \Rightarrow \quad |\vec{\tau}| = \mu B \sin(\pi / 2) = \mu B$$

$$|\vec{\tau}| = (1.72 \times 10^{-3} \text{ J / T})(0.350 \text{ T}) = 6.02 \times 10^{-4} \text{ N} \cdot \text{m}$$



Potential Energy of a Magnetic Dipole

- A potential energy can be associated to the magnetic dipole moment similar to the potential energy of an electric dipole moment.

$$U(\theta) = -\vec{\mu} \cdot \vec{B} = -|\vec{\mu}||\vec{B}|\cos\theta$$



Chapter 30

Sources of Magnetic Field



Magnetic Fields Produced by Currents

- When studying magnetic forces so far, we examined how a magnetic field, presumably produced by a permanent magnet, affects moving charges and currents in a wire.
- Now we consider the phenomenon in which a current carrying wire produces a magnetic field.
- Hans Christian Oersted first discovered this effect in 1820 when he observed that a current carrying wire influenced the orientation of a compass needle.

Biot - Savart Law

- Jean-Baptiste Biot and Fe'lix Savart formed this mathematical law based on a number of experiments that they carried out.

$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{I d\vec{s} \times \vec{r}}{r^3} = \frac{\mu_o}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$



Biot-Savart Law cont.

- The new constant in the Biot-Savart Law is called the permeability of free space and has a value of:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$$



Biot-Savart Law cont.

- The $d\mathbf{B}$ in the expression is only the magnetic field produced by a small segment of current in a conductor. To obtain the total B-field we need to integrate.



B-Fields around current carrying wires.

- The geometry of the magnetic field around a long straight current carrying wire is that of concentric circles that lie in a plane perpendicular to the direction of the current.
- The direction can be found by using a second right hand rule.
- Point your thumb in the direction of the current and curl your fingers inward. The direction that your fingers point is the direction of the magnetic field.



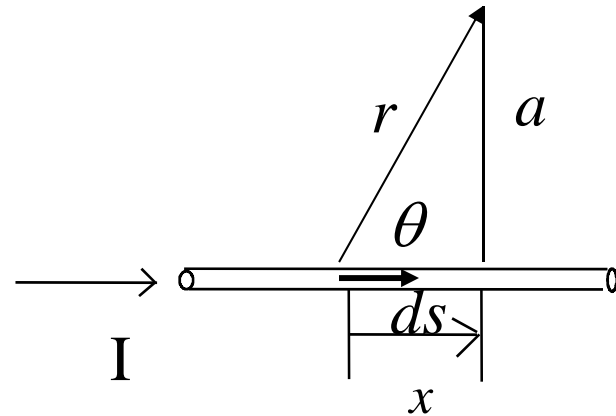
Example

- Using Biot-Savart Law, determine the magnetic field at a distance a away from a current carrying wire lying along the x axis.



Solution

- If we draw a picture of a small segment of the wire we can determine the geometry of the problem in terms of our equation.





Solution cont.

- If the \mathbf{k} direction is taken out of the page then when we take the cross product we obtain the following:



Solution cont.

- If we now substitute this expression into the law of Biot and Savart we get the following:



Solution cont.

- We now express our variables in terms of polar coordinates, since this problem is better dealt with using a cylindrical coordinate system.



Solution cont.

- Substitution into our equation yields:



Solution cont.

- Integration yields the total magnetic field around the wire at a distance a away.
- This result is good for any straight current carrying wire.



Solution for a Special Case

- Suppose in the previous example we let the current carrying wire become infinitely long.
- Then the angles go from zero to π .
- Our solution then looks like the following:



Example

- A long straight wire carries a current of 3.0A. A particle of charge $6.50 \mu\text{C}$ is moving parallel to the wire at a distance of 0.050m; the speed of the particle is 280 m/s.
- Determine the magnitude of the magnetic force exerted on the moving charge by the current in the wire.



Solution

- Since the particle is moving parallel to the wire and the magnetic field is always perpendicular to the current in the wire, then the particle's motion is perpendicular to the magnetic field.

$$F = qvB \sin \theta = qvB \sin(\pi / 2) = qvB$$



Solution cont.

- From our previous example we saw that the magnetic field around a long current carrying wire is:



Solution cont.

- If we plug this result into our equation for the force due to a magnetic field on a moving charge we get the following:



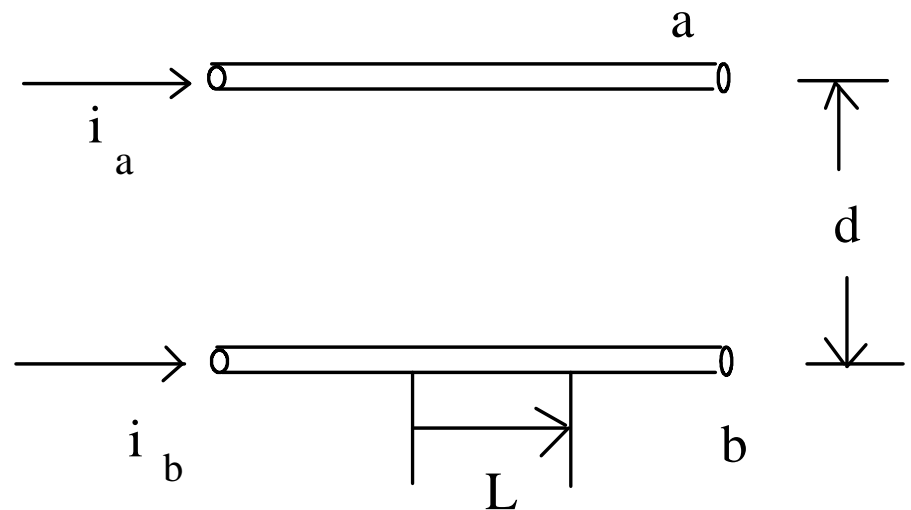
Solution cont.

- Plugging in our numbers from the problem we get the following result.

$$F = (6.5 \times 10^{-6} C)(280 m/s) \left[\frac{4\pi \times 10^{-7} T \cdot m / A(3A)}{2\pi(0.050m)} \right] = 2.2 \times 10^{-8} N$$

Two Parallel Currents

- Two long, parallel, current carrying wires exert a force on each other. The wires are separated by a distance d and have currents i_a and i_b respectively.





Two Parallel Currents cont.

- The force on wire b is produced by the magnetic field which is produced by the current in wire a .
- Biot-Savart law tells us that the magnitude of the magnetic field produced by wire a is given by:



Direction of B

- The direction of B_a can be obtained from the right hand rule. If the current is moving towards the right then the magnetic field due to wire a between the two wires is down.



Two Parallel Currents cont.

- The force on wire b due to a can be calculated from the expression for a force produced by a magnetic field on a current carrying wire.



Two Parallel Currents cont.

- The direction of \mathbf{B} , by use of the right hand rule, is toward wire a.
- Since \mathbf{L} and \mathbf{B}_a are perpendicular the magnitude of the force is given by:

Example

- The evil Goldfinger has captured James Bond and placed him on a torture device.
- The elaborate device is constructed from two large parallel, insulated, current-carrying cables that supply power to Goldfinger's mining equipment.



Example cont.

- The cables carry an equal amount of current but in opposite directions and they are separated by a distance of 3.0 m.
- Mr. Bond is lying on a table between the two cables with his hands secured to one cable while his feet are attached to the other.





Example cont.

- a) If cables are allowed to move freely and each has a length of 15 meters how much current must be sent through the cables to exert a force of 1000 N on James Bond?
- b) How does James escape from this device?



Solution a

- The forces exerted by one cable on the other will be equal but opposite in direction.
- Therefore, we need only to calculate the force of one wire on the other.



Solution a cont.

- The magnetic field created by the first cable can be calculated by the following:



Solution a cont.

- The force is then give by the following:



Solution a cont.

- Since both currents are equal and parallel with each other then our equation reduces to the following:



Solution a cont.

- Solving for the current yields:



Solution a cont.

- The amount of current in the cables that is required can now be determined.



Ampere's Law

- An alternative method for finding the magnetic field around a current carrying wire is the method developed by the French mathematician and physicist Andre-Marie Ampere.



Ampere's Law

- Ampere law: For any current geometry that produces a magnetic field that does not change in time:
- Here ds is a small increment around a closed loop that surrounds the current carrying wire and lies in the plane perpendicular to the wire.



Example

- A long straight current carrying wire produces a magnetic field that is radial to the wire.
- Use Ampere's law to determine the magnetic field around the wire at a distance r .



Solution

- The circular path is everywhere perpendicular to the current in the wire, therefore:



Solution cont.

- If we integrate around the Amperian loop we get the equation to the right.

- Solving for B yields:



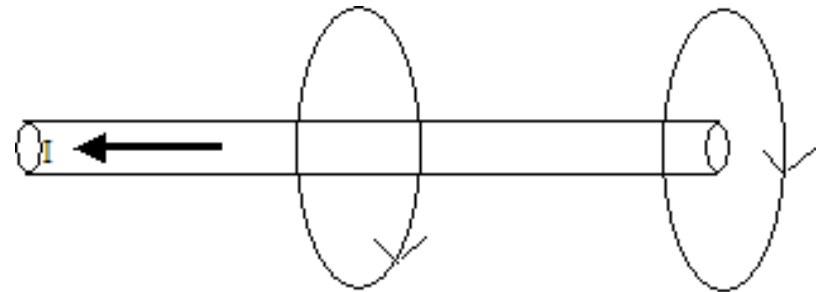
The B-Field Inside a Long, Straight Current carrying Wire

- The magnetic field in and around a long, straight current carrying wire is cylindrically symmetrically.
- To find the magnetic field inside the wire we use an Amperian loop of radius $r < R$, where R is the radius of the wire.



Diagram

- If the current is in the direction shown, then the right-hand rule gives the direction of the magnetic field as shown.





B-Field Inside Wire cont.

- The magnetic field is tangent to the loop and Ampere's law yields:



B-Field Inside Wire cont.

- The current enclosed by our loop is equal to the current density multiplied to the area of the loop.



B-Field Inside Wire cont.

- Then according to Ampere's law:



B-Field Inside Wire cont.

- After integrating around our Amperian loop and solving for the magnetic field we get the following:



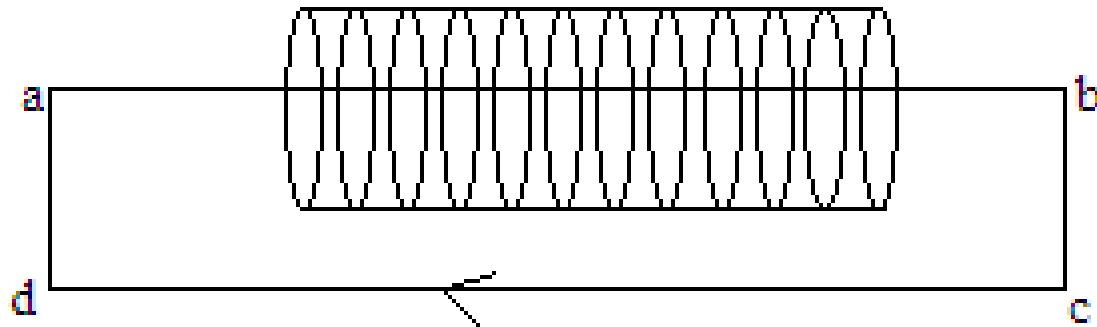
Magnetic Field of a Solenoid

- A solenoid is a long tightly wound helical coil of wire.
- If a current passes through the solenoid a magnetic field is produced.
- If the solenoid's length is much greater than its radius then the magnetic field outside of the solenoid is essentially zero.



Diagram

- The Amperian loop for the solenoid passes through the coil on the inside and outside as shown.





Magnetic Field of a Solenoid

cont.

To find the magnetic field inside the solenoid we apply Ampere's law.

If we make a rectangular Amperian loop such that it encloses N number of turns of the wire Ampere's law give the following result:

Magnetic Field of a Solenoid

cont.

- The only part of the path integral that is non-zero is the path from a to b, where the magnetic field is parallel to the path ds . Thus:

Magnetic Field of a Solenoid

cont.

- Therefore, the magnetic field for an ideal solenoid is:

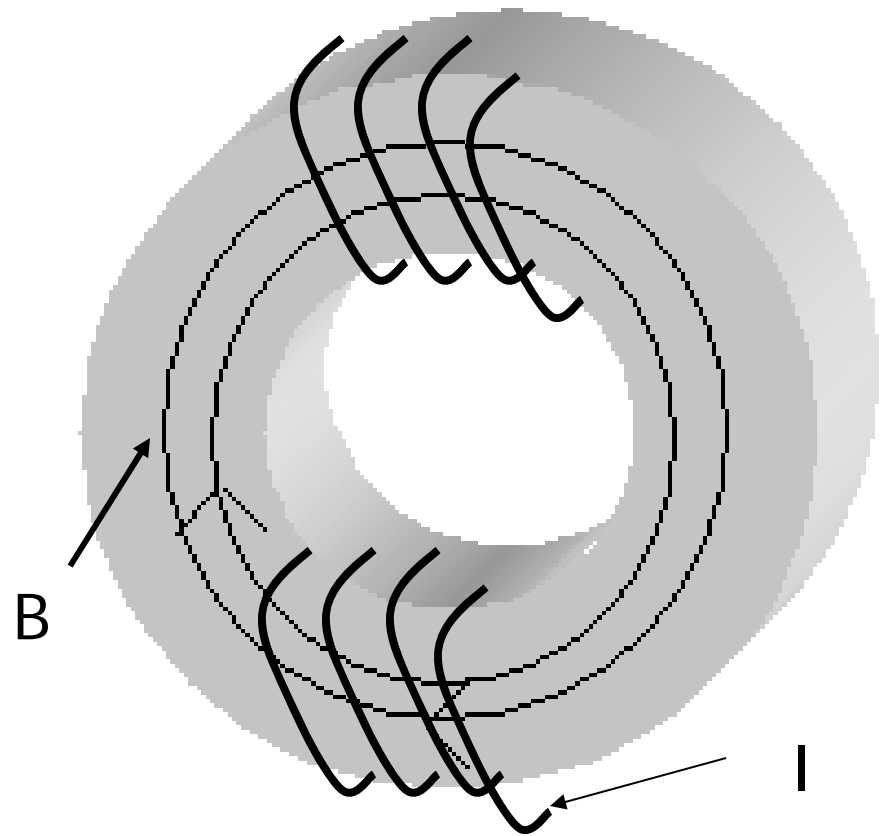


Magnetic Field of a Toroid

- A toroid is formed when a solenoid is bent around into a doughnut shape.
- For an ideal toroid the magnetic field outside is zero.

Magnetic Field of a Toroid

- A cross section view of a toroid shows that the magnetic field lines form concentric circles.





Magnetic Field of a Toroid

- If we choose our Amperian loop to be a circle within the toroid with a radius r , then Ampere's law yields: